$(n,2n)$ Reactions in C^{12}, Cu^{63}, and Mo^{92}†

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Using the method of induced radioactivity we have measured the $(n,2n)$ cross sections of C^{12}, Cu^{63}, and Mo^{92} as a function of energy from threshold to 27 Mev. The positron activities of 20, 10, and 15.5 min, respectively, were employed. K-capture corrections were applied to the copper and molybdenum data. We have compared the copper and molybdenum data with the statistical theory of Weisskopf and collaborators and find satisfactory agreement from threshold to the onset of tertiary reactions.

INTRODUCTION

THE statistical theory[1,2] gives an account of $(n,2n)$ reactions for nuclei with mass number greater than about 50, in terms of a rather simple model. If the variation of cross section with energy for this type of reaction is known, deductions about the effective energy level densities in the nuclei concerned can be made. Radioactivities produced by the $(n,2n)$ reactions are frequently employed as fast neutron detectors. For these reasons we have investigated $(n,2n)$ cross sections as a function of energy for C^{12}, Cu^{63}, and Mo^{92}. The associated half-lives are of convenient duration and K-capture corrections are small. In the case of Cu^{63} this study extended the range of measurements which have been performed at lower energies.[3–5] The cross section of C^{12} has been measured with 90-Mev neutrons.[6]

EXPERIMENT

The source of neutrons was the $T(d,n)He^4$ reaction. Tritium gas contained in a thin-walled target cell was bombarded by 10.5-Mev deuterons from the 42-in. cyclotron. The energy of the neutrons produced at different angles to the deuteron beam can be calculated from the measured deuteron energy and the known energy release from the $T(d,n)He^4$ reaction, 17.6 Mev. A description of the target cell and alignment procedure has been given in a paper on (n,p) scattering at 27 Mev.[7]

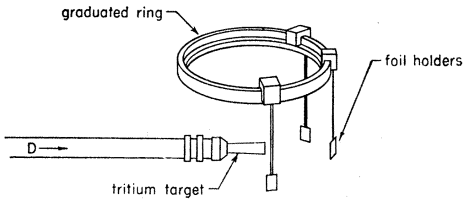

FIG. 1. Arrangement of foils around the tritium target.

The absolute differential cross section as a function of angle for the reaction $T(d,n)He^4$ has been measured by counting the neutrons and α-particles; results are given in the paper by Brolley, et al.[8] The samples, which in the case of copper and molybdenum were enclosed in thin cadmium jackets were supported around the target cell in such a way that there was negligible scattering material in the vicinity of the target. The arrangement is schematically shown in Fig. 1. The general bombardment and counting techniques have been discussed in an earlier report.[4]

RESULTS

In the curves of experimental data (Figs. 2–4) the total spread in energy of the neutrons intercepted by the foils is indicated by horizontal lines. It is calculated from the angular resolution as well as from the spread in energy resulting from the slowing down of the deuterons in the tritium target.

$C^{12}(n,2n)C^{11}$

The threshold of this reaction as calculated from mass tables[9] and taking into account the recoil energy of the carbon nucleus is 20.2 Mev. C^{11} decays by positron emission with a half-life of 20.3 min.[10] The samples irradiated were in the form of polyethylene foils about 11 mg/cm^2 thick. A lamina of two thicknesses of polyethylene was irradiated and counted as a single foil.

Since the end-point energy of the positron emitted by C^{11}(0.95–0.98 Mev)[10] is approximately the same as the end-point energy of the beta-particles from the RaD+E National Bureau of Standards source (1.17 Mev), it is convenient to calibrate the Geiger counters by use of the RaD+E Standard. Due to the low intensity of the C^{11} activity the following experimental procedures were adopted.

Rectangular foils, of polyethylene 2.54×5.08 cm, were folded into a 2.54-cm square and supported about 15 cm away from the center of the target cell. Similar 2.54×5.08 cm foils were folded and placed between two circular foils 1.43 cm in diameter (the diameter of the radioactive layer of the RaD+E standard). This

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arrangement of foils was fastened directly on the end of the target cell. After the bombardment the circular disks were counted with end-window counters with the same geometry and with the same type of silver backing as supported the RaD+E source. The rectangular foils were all counted on a set of three glass-walled Geiger counters. Since the activity per unit mass of all the foils next to the target cell was the same, the rectangular foils activated in juxtaposition with the disks could be used to calibrate the cylindrical counters in terms of the end-window counters, which in turn were calibrated against the RaD+E standard.

In order to compensate to some extent for the effects of self-absorption and scattering in the sample, a single thickness of inactive polyethylene was placed over the RaD+E source during a calibration run. A relative correction of 7.5 percent had to be applied in the efficiency calculations due to the difference in energy of the beta-particles from C^{11} and those from the RaD+E

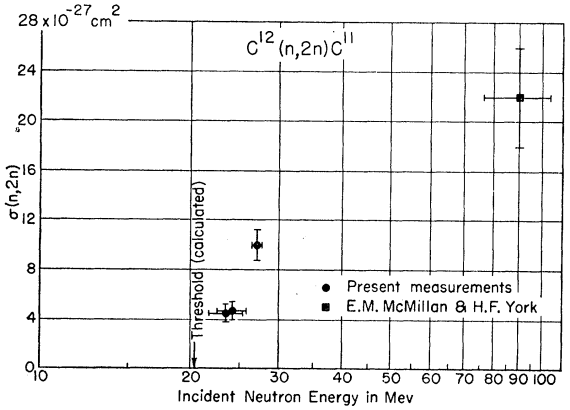

FIG. 2. Cross section of the $C^{12}(n,2n)C^{11}$ reaction as a function of neutron energy. The ordinate represents the absolute cross section for the production of the 20.3 minute positron emitter C^{11} from the C^{12} isotope. The data has not been corrected for K-capture competition.

standard and due to the difference of thickness of polyethylene involved in the two different cases. This correction was calculated from the absorption coefficients as well as effects resulting from fore- and backscattering estimated from the measurements of Zumwalt.[11] As a check on these calculations, a curve was taken of the counting rate of an end-window counter as a function of thickness of polyethylene placed over the RaD+E standard, which was 2.8 cm away from the counter window. The curve calculated by using Zumwalt's measurements of self-absorption, self-scattering, and backscattering effects agreed fairly well with the measurements. The 8 percent correction to the value of the RaD+E standard found necessary by Burtt[12] was applied to the data.

Figure 2 gives the absolute cross section for production of C^{11} by the ($n,2n$) reaction as a function of the energy of the incident neutrons. The circles denote the present measurements. The vertical spread indicated is the estimated standard error of the measurements. Besides the standard deviations from the mean of a number of measurements, the errors indicated in Fig. 2 also include the errors in the neutron counting (11 to 15 percent) as well as errors involved in the absolute beta-counting, which were estimated to be about 6 percent. The values plotted have been corrected for the isotopic abundance of C^{12} in normal C. For comparison, the reported value at 90 Mev is included in Fig. 2.[6] For this point the horizontal spread is quoted as the width of the neutron energy distribution at half-maximum. The cross sections in Fig. 2 have not been corrected for K-capture competition with positron emission and therefore represent only the cross section for production of the positron activity.

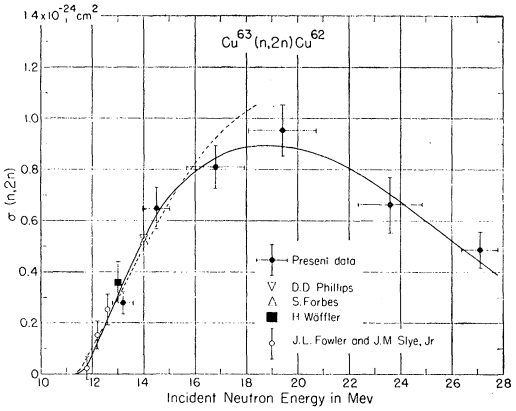

FIG. 3. The cross section of the $Cu^{63}(n,2n)Cu^{62}$ reactions as a function of neutron energy. The data has been corrected for isotopic abundance of Cu^{63} and for K-capture competition with positron emission. The broken line gives the predictions of the statistical theory, ignoring competitive proton emission.

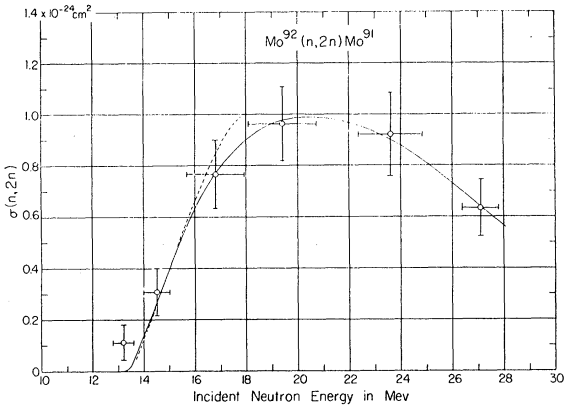

FIG. 4. The cross section of the $Mo^{92}(n,2n)Mo^{91}$ (15.5-minute period) reaction as a function of neutron energy. The data has been corrected for isotopic abundance of Mo^{92} as well as K-capture competition with positron emission. The broken line gives the prediction of the statistical theory, ignoring competitive proton emission.

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$Cu^{63}(n,2n)Cu^{62}$

The threshold of this reaction calculated from reported values of (γ,n) thresholds as well as $(n,2n)$ thresholds[10,13] is 11.1±0.2 Mev. Cu^{62} decays with positron emission of maximum energy 2.8–2.9 Mev and period of 10 min.[10] The samples irradiated were Cu foils 2.54 cm×5.08 cm and 0.0127 cm thick. The data were made absolute in terms of the known thermal capture cross section of Cu^{65} by calibrating the Geiger counters with the beta-particles from Cu^{66} produced from Cu^{65} by thermal neutrons.[4] A recent and somewhat more accurate measurement of the $Cu^{65}(n,\gamma)Cu^{66}$ cross section reduces the uncertainty in this type of calibration over that given in reference 4.[14] Cu^{66} decays with the emission of electrons of maximum energy 2.6–2.9 Mev.[10]

The results for the $Cu^{63}(n,2n)Cu^{62}$ cross section are shown in Fig. 3, together with results of other investigators. The points of previous experiments due to Phillips and to Fowler and Slye[4] have been corrected for the new value of the capture cross section in Cu^{65}. All of the points in Fig. 3, with the exception of the value due to Wäffler,[5] have been corrected by us for K-capture competition with positron emission. This correction raises the points approximately 1.7 percent in value and was calculated from Feenberg and Trigg's[15] curves with the end-point energy of the positron taken as 2.9 Mev. Wäffler had already applied a K-capture correction to his measurement.

The standard errors indicated by the vertical lines are estimated from the standard deviations of a number of runs, the uncertainty in the absolute beta-counting (about 10 percent), and the uncertainties in the measurement of the neutron flux. It is apparent that the various measurements are consistent with each other within the limits of error.

The statistical theory of Weisskopf and collaborators[1,2] should apply to copper; we have therefore computed the theoretical $(n,2n)$ cross section for Cu^{63} using the value $a=2.2$ Mev^{-1} [nuclear temperature $=(E/2.2)^{\frac{1}{2}}$], suggested by Feld *et al.*[16] The theoretical cross section is indicated by the dashed curve in Fig. 3. The agreement with experiment is good from the threshold up to 16 Mev. At 18 Mev it is rather higher than the experimental curve but still consistent. In the neighborhood of 18-Mev tertiary reactions compete and therefore the $(n,2n)$ cross section will drop.

It should be noted that the theoretical calculation we have used takes no cognizance of competitive proton emission. This process may well occur in the second emission. From considerations of reactions and beta-ray kinetics, as well as from theoretical mass calculations, a proton binding energy of about 6.5 Mev in Cu^{63} may be inferred. Since this is significantly lower than the binding energy of a neutron to Cu^{63}, some proton emission might be expected from Cu^{63} in its highest excitation states.

$Mo^{92}(n,2n)Mo^{91}$

The threshold of the $Mo^{92}(\gamma,n)Mo^{91}$ reaction has been measured as 13.28±0.15 Mev[17] and 13.1±0.1 Mev.[18] An earlier direct measurement of the $(n,2n)$ threshold for the production of the 15.5-min period gave a value between 12 and 13 Mev.[19] The average of these measurements, weighted according to their errors, the (γ,n) threshold being corrected for the recoil energy for the case of $(n,2n)$ reactions, gives the $(n,2n)$ threshold as 13.2±0.1 Mev. Mo^{92} is a magic number nucleus with a closed shell of 50 neutrons. The Mo^{91} isotope has two isomeric states and decays with two periods: One has a half-life of 15.5 min and emits a positron with end-point energy of about 3.3 Mev; the other has a half-life slightly over one minute (75 sec,[20] 65.5 sec[18]) and emits a positron with end-point energy of about 2.6 Mev.[17,18,20]

The sample in the case of this element consisted of foils of normal molybdenum 2.54×5.08 cm and 0.0127 cm in thickness. For obtaining relative values of the cross section, these foils were counted on three glass-walled cylinder type Geiger counters. The shorter period reported for Mo^{91} produced by betatron irradiation was not observed in this experiment.[18,20] In order to normalize the cross section to absolute values, 1-cm diameter disks of copper and molybdenum were irradiated simultaneously in juxtaposition with neutrons coming off in the forward direction from the $(D+T)$ reaction, and the activities produced in these disks were compared by use of an end-window Geiger counter. By extrapolating the measurements of Zumwalt[11] in the manner which he suggests, one can make an estimate of the relative effects of beta-ray absorption and self-scattering. When this was done it was found that the relative efficiencies for counting were the same within one percent. The range of extrapolation, however, was such that one concludes the estimate of the relative efficiency of counting may be in error as much as 7 percent

Thus, the cross section of the $Mo^{92}(n,2n)Mo^{91}$ reaction for neutrons in the forward direction was calculated in terms of the cross section of the $Cu^{63}(n,2n)Cu^{62}$ reaction for these neutrons.

Figure 4 gives the results of the $Mo^{92}(n,2n)Mo^{91}$ measurements. The data have been corrected for K-capture competition with positron emission (4.5 percent) assuming an end-point energy of the positron 3.3 Mev. The uncertainties indicated by the vertical spread are estimated from the reproducibility of measurements, the uncertainty in determining the molybdenum cross

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sections relative to the copper cross sections, the uncertainty in the copper cross section, and the uncertainty of the neutron measurements.

The dashed curve in Fig. 4 is a theoretical calculation of the $Mo^{92}(n,2n)Mo^{91}$ cross section based on the statistical theory[1,2] with $a=3.1$ Mev^{-1} (nuclear temperature$=(E/3.1)^{\frac{1}{2}}$ Mev). The value $a=3.1$ Mev^{-1} is reasonable for this region of the periodic table. It was chosen to give the best fit to the data. As in the case of copper, agreement with the statistical model is satisfactory up to about 18 Mev, where tertiary reactions began to compete.

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The Impulse Approximation and Field Theoretical Calculations. I

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The reaction α+nucleus$\rightarrow\beta$+nucleons is considered where α and β are particles of quantized fields and the coupling to nucleons is assumed linear in the fields. By an extension of the recent work of Chew and Goldberger on potential scattering, the collision matrix has been manipulated into a term representing the impulse approximation plus three correction terms which are, respectively: The error in the impulse assumption arising from nuclear potentials; the "multiple scattering" which is at least fourth order in the coupling; and finally, a small term representing the effect of lowest order absorption. It is significant that there is no large term present, of the order of the impulse approximation term, which represents processes, second order in the coupling, where one nucleon absorbs α and another nucleon emits β.

I. INTRODUCTION

THE impulse approximation, introduced by Chew[1] in a study of the inelastic scattering of neutrons by the deuteron, has proved useful for a variety of nuclear problems. The essence of this approximation is that the scattering amplitude for a nucleus can be written as the sum of the free scattering amplitudes of the single nucleons whose momentum distribution is determined by the initial nuclear state function. The conditions under which the impulse approximation can be expected to hold were stated first by Chew and Wick[2] and recently were more rigorously demonstrated by Ashkin and Wick[3] and Chew and Goldberger[4] for the case of potential scattering.

These conditions are[4] (A) the incident particle interacts only with one particle at a time; (B) the amplitude of the incident wave is not appreciably reduced in crossing the nucleus; (C) the nuclear binding potential U has a negligible effect during the interval of strong interaction. Condition (C), the so-called "impulse assumption," was shown equivalent[2] to the requirement that the "collision time" τ be short compared to a time characteristic of the nuclear binding $(1/U)$.

The applications of the impulse approximation have not been limited to potential scattering; they have also been extended to field theoretical calculations such as photomeson production[5–8] and meson scattering.[9–14] It has been qualitatively argued[9,12,2] that condition (C) is satisfied for the case of meson-deuteron scattering as treated by conventional weak coupling theory, but a more quantitative argument justifying the impulse approximation in a field theoretical calculation has not been published. To repeat the qualitative argument: The reciprocal of the "collision time" $(1/\tau)$ can be equated to the amount by which energy conservation is violated in the intermediate state; this energy violation is of the order of the total meson energy, ω, which is large compared to the nuclear potential U. It is not immediately obvious, however, that this qualitative argument implies that the term in the collision matrix will vanish, which represents, to second order in the coupling, absorption of a field particle by one nucleon and emission by another nucleon.

The purpose of this note is then to discuss formally the impulse approximation when applied to field theoretical calculations such as meson scattering; the task

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